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CALCULATION OF TRANSIENT PROCESSES IN PHASE-CONTROLLED DISCRETE SYSTEMS USING THE MATLAB 6.0 SOFTWARE

M. P. Batura and D. A. Gan'shin

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The issues of investigation and analysis of phase-controlled discrete systems are considered. Two methods of modeling (piecewise analytical and simulation) are proposed. The characteristic properties of the implementation of the simulation method are analyzed from the viewpoint of the use of the MATLAB 6.0 software package for automation of engineering calculations.

Phase-controlled discrete systems, operating at high quantization frequencies, show the highest precision characteristics as compared to many other automatic devices. Owing to this they have gained wide acceptance in communications, radioengineering, and teleautomatics facilities. However the presence of substantially nonlinear dynamic elements in the structure of a phase-controlled discrete system makes it impossible to analyze and accurately calculate it without a personal computer. It must be noted that selection of software is of importance here.

We dwell on the problem of modeling of a phase-controlled discrete system. We give three approaches to its solution, two of which rely on the computer powers of the MATLAB system of automation of engineering calculations.

We consider the simplest model of a phase-controlled discrete system. Its structural scheme is given in Fig. 1. Here the simplest trigger is employed as the discrete comparator (DC). The driving action of the system in question is the pulse train $y(t)$. The pulse train $u(t)$ is fed from the output of the frequency divider with a division coefficient K to the other input of the discrete comparator. The mismatch signal $\varepsilon(t)$, which is a train of width-frequency-modulated pulses (width-modulated ones if the frequency is f_0 = const), arrives at the controlled element of a tunable generator via a low-pass filter.

The frequency at the output of the tunable generator is determined by the expression

$$w(t) = g(t) + z(t) . \tag{1}$$

The equality of the frequencies f_0 and f_{fb} of the signals $y(t)$ and $u(t)$ is observed in the phase-controlled discrete system in the synchronization regime.

The law of pulse-width modulation in the synchronization regime is described by the expression

$$K \int_{t_n+\tau_n}^{t_n+T_n+\tau_{n+1}} w(t)\, dt = 2\pi . \tag{2}$$

Integration of the frequency between the limits from $t_n + \tau_n$ to $t_n + T_n + \tau_{n+1}$ yields the phase incursion of the feedback signal over the period T_n equal to 2π. The pulse-width modulation described by expression (2) belongs to the class of integral, or threshold, pulse-width modulation.

Let the continuous linear part of the system be described by equations of state of the form

$$\dot{X}(t) = AX(t) + B\varepsilon(t) , \quad z(t) = CX(t) + D\varepsilon(t) . \tag{3}$$

Belarusian State University of Informatics and Radioelectronics, Minsk, Belarus; email: Hanshyn@hotmail.com. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 76, No. 2, pp. 193–195, March–April, 2003. Original article submitted December 19, 2002.

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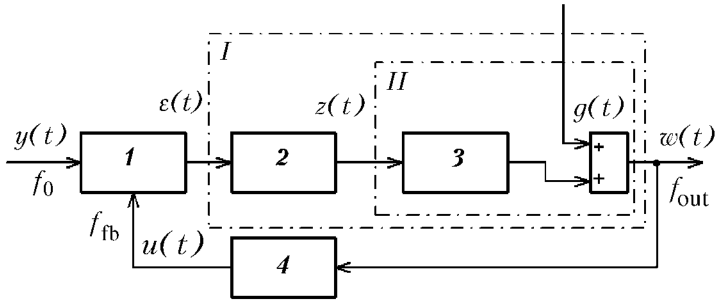

Fig. 1. Structural scheme of the phase-controlled discrete system: I, continuous linear part; II, tunable generator; 1) discrete comparator; 2) low-pass filter; 3) controlled element; 4) frequency divider with a division coefficient K.

We denote the vector of state at the instant of time $t = t_n$ by X_n; then its change on the intervals of action of the pulses and the pause is described by the equations

$$X(t) = H(t - t_n)X_n + \int_{t_n}^{t} H(t-u)\,Bh\,du\,, \quad t \in [t_n, t_n + \tau_n]\,; \tag{4}$$

$$X(t) = H(t - t_n)\,X_n + \int_{t_n}^{t_n+\tau_n} H(t-u)\,Bh\,du\,, \quad t \in [t_n + \tau_n, t_{n+1}]\,; \tag{5}$$

$$X(t) = H(t - t_n)X_n + \int_{t_n}^{t_n+\tau_n} H(t-u)\,Bh\,du + \int_{t_{n+1}}^{t_n} H(t-u)\,Bh\,du\,, \quad t \in [t_{n+1}, t_{n+1} + \tau_{n+1}]\,. \tag{6}$$

Setting $t = t_{n+1}$ in (5), we obtain the equation relating the vector of state at discrete instants of time:

$$X(t) = H(T_n)\,X_n + \int_{t_n}^{t_n+\tau_n} H(t_{n+1} - u)\,Bh\,du\,. \tag{7}$$

Since $H(t) = \exp(At)$, we, carrying out the operation of integration, obtain

$$X_{n+1} = \exp(AT_n)\,X_n + A^{-1}\exp(AT_n)\,(E - \exp(-A\tau_n))\,Bh\,. \tag{8}$$

Expression (8) represents the nonlinear difference equation of the continuous linear part in the case of combined action of pulse-width-frequency modulation.

We substitute $w(t) = z(t) + g(t)$ into (2) and, with account for (3), (5), and (6), obtain the equation of closure of the system:

$$\begin{aligned} &C\Big\{A^{-1}[\exp(A(T_n + \tau_{n+1})) - \exp(A\tau_n)]\,X_n + A^{-2}[\exp(A(T_n + \tau_{n+1})) - \\ &- \exp(A(T_n + \tau_{n+1} - \tau_n)) - \exp(A\tau_n) + \exp(A\tau_{n+1})]\,Bh - A^{-1}\tau_{n+1}Bh\Big\} + \\ &+ Dh\,\tau_{n+1} + g_n(T_n - \tau_{n+1}) + g_{n+1}\,\tau_{n+1} = 2\pi/K_2\,. \end{aligned} \tag{9}$$

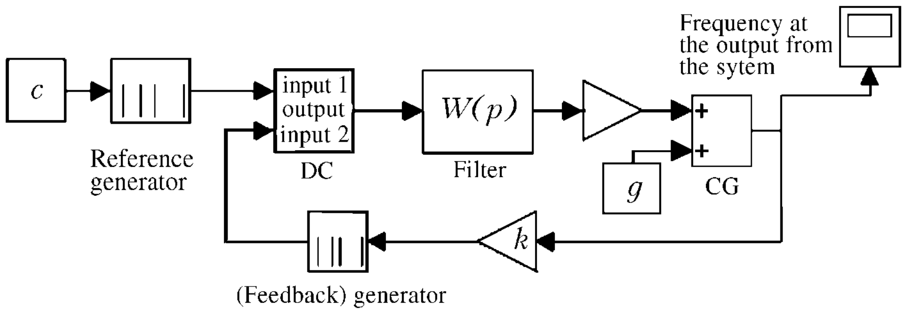

Fig. 2. Model of the phase-controlled discrete system in the Simulink environment: $W(p)$, transfer function of the low-pass filter; K_g, gain of the controlled generator (G); g, eigenfrequency of the controlled generator; c, frequency of the reference generator.

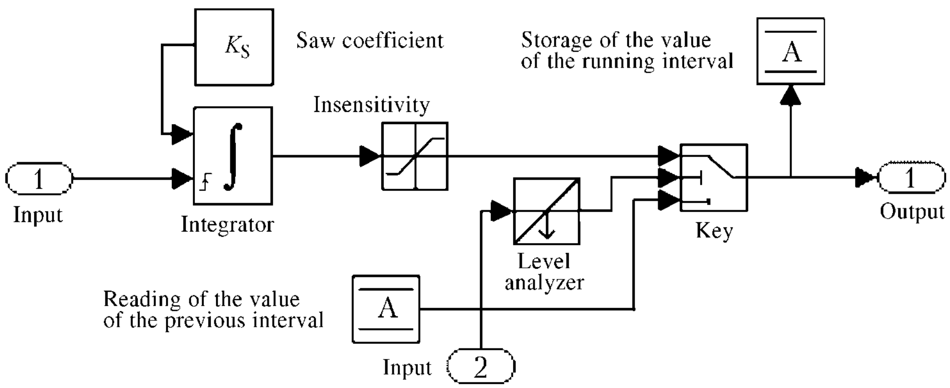

Fig. 3. Discrete-comparator block: K_s, sweep coefficient of the saw.

Here g_n = const at $t \in [t_n + \tau_n, t_{n+1}]$ and g_{n+1} = const at $t \in [t_{n+1} + \tau_n, t_{n+2}]$.

Equations (8) and (9) represent the nonlinear discrete model of the system with a trigger discrete comparator which makes it possible to recurrently determine the vector of state variables and hence to calculate the transient process in the system. Thus, when X_n and τ_n are known, we calculate X_{n+1} from Eq. (8), τ_{n+1} from Eq. (9), etc. [1].

We consider the approach constructed on the simulation principle. By a simulation mathematical model we mean an algorithmic model reflecting the behavior of the object under study with time when the external actions on it are prescribed. This model enables one to calculate the processes occurring in the phase-controlled discrete system in the time domain, to derive the values of any variables at arbitrary instants of time, and to interactively replace comparators and change the values of individual variables. The piecewise-analytical method presents one with such possibilities mainly through total revision of mathematical expressions.

In [2], the simulation model has been developed in the form of a program in the MATLAB language. Each block of the system was described by an individual function. The principal module (procedure) combined all the functions into a single system of equations, determining the general rules of interaction of the blocks with each other and with the user. The entire amount of information on the processes was stored in matrix form and could be displayed or subjected to additional analysis at any instant. Although such a model ranked below the previous models in efficiency, it was more versatile and convenient in operation. A change in the structure of the system (for example, of the type of discrete comparator) involved a change of only the corresponding function now.

Dealing with the issues of simulation modeling, it would be wrong to exclude from consideration the Simulink visual-modeling facility built into the MATLAB. Vast Simulink libraries contain a large set of ready-made blocks using which one can construct models of rather complex systems. Thus, the model of a phase-controlled discrete system with a discrete comparator of the "retrieval–storage" type which has been implemented with the Simulink facilities has the form presented in Fig. 2.

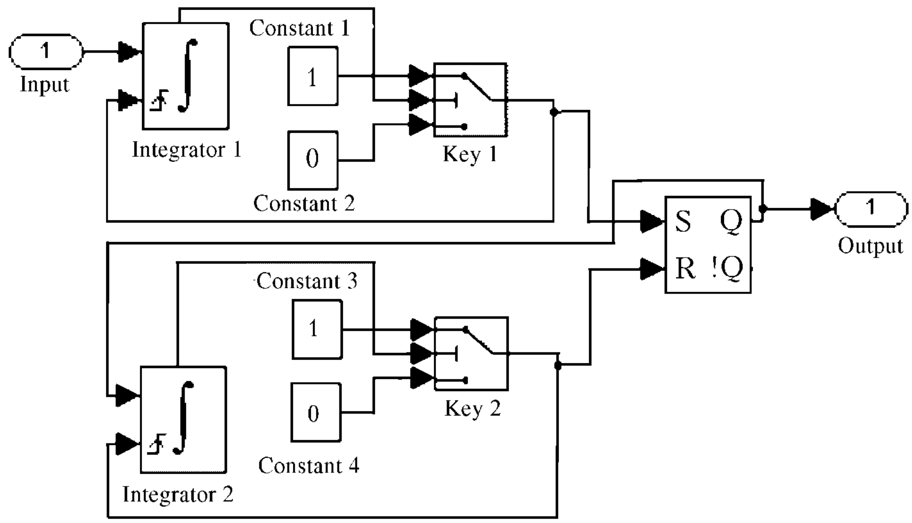

Fig. 4. Tunable-generator block: *S* and *R*, set and reset inputs of the trigger; *Q* and !*Q*, direct and inverse outputs of the trigger.

The blocks of the generator and the discrete comparator are composite. Each of them represents an individual model (Figs. 3 and 4). The advantage of such a model is clear representation and convenience in operation. Using the built-in facilities one can track a change in the signal at any point and save the information in a file. Among the drawbacks are the lower speed as compared to the previous models (8) and (9).

Each of the above approaches to studying the transient processes in phase-controlled discrete systems has its undeniable advantages and certain drawbacks. One can always make a choice in favor of one of the presented approaches in accordance with the special properties of the problem posed. But the best result will be obtained when these approaches are employed in combination.

NOTATION

$u(t)$, train of pulses from the output of the frequency divider; K, division coefficient of the frequency divider; $g(t)$, frequency of the stabilized tunable generator; $\varepsilon(t)$, mismatch signal from the output of the discrete comparator; f_0, frequency at the output of the reference generator; f_{fb}, frequency at the output of the frequency divider (feedback); $w(t)$, frequency at the output of the controlled generator; $z(t)$, frequency change introduced by the control element into the generator; t_n and t_{n+1}, time instants of the arrival of the nth and nth + 1 pulses, respectively; τ_n and τ_{n+1}, duration of the nth and nth + 1 pulses; T_n and T_{n+1}, periods of the signals on the nth and nth + 1 intervals; A, B, C, and D, matrices of description of the continuous linear part; X_n and X_{n+1}, vector of state of the continuous linear part on the nth and nth + 1 intervals; E, unit matrix; $H(t)$, transient characteristic; h, pulse height. Subscripts: fb, feedback; s, saw; out, output; g, generator;

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